STUDENT NUMBER: ________________

STUDENT NAME: __________________

THE HILLS GRAMMAR SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

2008

MATHEMATICS

2 UNIT ADVANCED

Teacher Responsible: Mr K McClure

General Instructions:

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen.
- Marks may be deducted for careless, untidy or badly arranged work.
- Board approved calculators may be used.
- A table of standard integrals is supplied at the back of this paper.
- ALL necessary working should be shown in every question.
- **Start each question in a new booklet.**

Total marks – 120

- Attempt Questions 1-10
- All questions are of equal value.

Students are advised that this is a Trial Examination only and cannot in any way guarantee the content or the format of the Higher School Certificate Examination.

Question 1 (12 Marks) **Marks**

a) Solve the equation

$$4(x-2)-3(x-5)=17$$ **2**

b) Calculate the sum to infinity of the series

$$2+1+\frac{1}{2}+\frac{1}{4}+$$ **2**

c) Solve the equation $2x^2+7x-15=0$ **2**

d) Find the perpendicular distance of the point $(10,-2)$ from the line $3x+4y-7=0$ **2**

e) Rationalise the denominator of $\dfrac{1}{2\sqrt{2}-1}$ **2**

f) The sector of a circle of radius 4 cm subtends an angle of $\dfrac{5\pi}{18}$ radians at the centre. Find the arc-length subtended by this angle.

Give your answer to 1 decimal place. **2**

Question 2 (12 Marks) **Marks**

a) Differentiate with respect to x and fully simplify your answer.

i) $y = x^2 \, e^x$ **2**

ii) $y = \dfrac{\sin x}{\cos x}$ **2**

b)

i) Find $\displaystyle\int \frac{x^2 + 1}{x^3 + 3x} \, dx$ **2**

ii) Find the area under the curve $f(x) = \dfrac{2}{1-3x}$ from $x = 2$ to $x = 1$

Leave your answer in exact form. **3**

c) The equation of the curve C is given by $y = \dfrac{4}{x}$. Find the equation of the tangent to the curve C at the point $P(1, 4)$ **3**

Question 3 **(12 Marks)** **Marks**

a) i) Plot the points A (3, 3) B (8, 0) C (−1, 1) and D (−6, 4) **1**

ii) Find the co-ordinates of the point of intersection of the lines AC and BD. **4**

b) The first four terms of an arithmetic progression are 5, 11, 17 and 23.

Find i) the 30th term **1**

ii) the sum of the first 30 terms. **2**

c) The sum to infinity of a geometric progression is 7 and the sum of the first two terms is $\frac{48}{7}$.

i) Show that the common ration, r, satisfies the equation $1 - 49r^2 = 0$ **2**

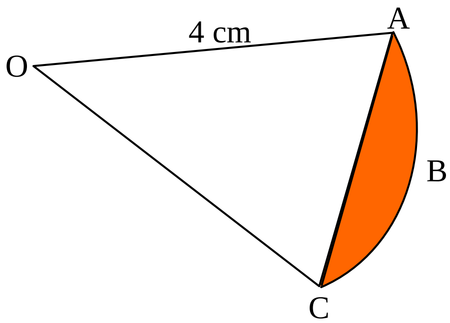
ii) Find the first term of the geometric progression with a positive common ratio. **2**

Question 4 (12 Marks) **Marks**

a) Solve $\cos^2\theta = \frac{1}{2}$ for θ where $0° \le \theta \le 360°$ **3**

b) Solve the equation $x^4 + 5x^2 - 14 = 0$ **3**
(hint, let $m = x^2$)

c)



The diagram shows part of a circle, centre O and radius 4 cm. Given that the arc length ABC is 5 cm, calculate

i) The size of angle AOC in radians **2**

ii) The area of the shaded region **2**

d) Find the equation of the tangent to the curve $f(x) = \frac{1}{x^2}$
at the point $P(-1, 1)$ **2**

Question 5 (12 Marks) **Marks**

a) Find the values of the constant k given that the equation $(5k+1)x^2 - 8kx + 3k = 0$ has equal roots. **3**

b) Find the maximum area of a rectangle which has a perimeter of 28 units by first constructing a formula for area (A) in terms of the length of one side (x). **3**

c) Determine the intervals for which the function $f(x) = x^4 - 8x^2 - 3$ is increasing and decreasing. **3**

d) Find the area enclosed between the curve $y = x^2 - 2x - 3$ and the line $y = x + 1$ **3**

Question 6 **(12 Marks)** **Marks**

a) Solve the equation $3^x = 10$ for x (to 1 decimal place). **2**

b) Consider the curve given by $f(x) = x^4 - 4x^3$.

i) find the stationary points **2**

ii) determine the nature of the stationary points **2**

iii) find any points of inflexion **1**

iv) sketch the curve clearly showing all relevant points. **2**

c) Find the centre and the radius of the circle $x^2 + y^2 + 2x - 4y - 4 = 0$ **3**

Question 7 **(12 Marks)** **Marks**

a) i) Express $y = x^2 + 6x + 6$ in the form $(x-h)^2 = 4a\,(y-k)$ **2**

ii) state the vertex **1**

iii) state the focus **1**

iv) find the equation of the directrix **1**

b) The area enclosed between the curve $y = 4 - x^2$ and the line $y = 4 - 2x$ is rotated through 2π radians about the x-axis. Find the volume of the solid generated, leave your answer in terms of π. **4**

c) Consider the equation $y = 3\cos 2\theta$ for θ where $0° < \theta < 360°$

i) state the amplitude of the curve **1**

ii) find the period of the curve **1**

iii) Make a neat sketch of the curve **1**

Question 8 (12 Marks) **Marks**

a) Find the equation of the normal to the curve $y = \ln\left(\dfrac{x-1}{x+1}\right)$ at the point P where $x = 3$ **3**

b) Using Simpson's rule with 5 ordinates, find an approximation for the area under the curve $f(x) = e^{-2x}$ between $x=1$ and $x = 3$ **3**

c) Solve the equation $4\cos^2\theta + 3\sin\theta = 4$ where $0° \le \theta \le 360°$ **3**
(Hint: let $\cos^2\theta = 1-\sin^2\theta$)

d) Find $\dfrac{dy}{dx}$ given that $y = \sqrt{6+x}\sqrt{3-2x}$ **3**

Question 9 (12 Marks) **Marks**

a) A particle moves in a straight line and at any time t seconds $(t \geq 0)$, its displacement from a fixed point 0 on the line is given by $x(t) = 4 + 2\cos\frac{\pi}{5}t, \quad 0 \leq t \leq 20$

i) Sketch the graph of the displacement against time. **3**

ii) Find when the velocity of the particle is zero. **3**

b) The population $P(t)$ of persons in a new suburb increases at a rate given by the equation $\frac{dP}{dt} = kP$, where k is a constant

and t is the time in years. The population of the suburb is expected

to double every fifteen years.

It may be assumed that $P = P_0e^{kt}$ where P_0 is the initial population.

i) Find the value of k .Leave your answer in exact form. **2**

ii) In which year will the suburb attain a population four times that which it had at the beginning of the year 2000? **2**

iii) It is known the population of the suburb at the beginning of the year 2000 was 30,000. What is the population expected to be in the year 2035? **2**

Question 10 (12 Marks) **Marks**

a) A piece of wire of length 1 m is cut into two parts and each part is bent to form a square. If the total area of the two squares formed is 325 cm^2 , find the perimeter of each square. **6**

b) A closed, right circular cylinder of base radius r cm and height h cm has a volume of 54π cm^3.

i) Show that S, the total surface area of the cylinder is given by $S = \frac{108\pi}{r} + 2\pi r^2$ **3**

ii) Hence find the radius and height which make the surface area minimum. **3**

END OF EXAMINATION

Q1 a) $4x-8-3x+15=17$

$x+7=17 \qquad x=10$

b) $a=2,\ r=\frac{1}{2} \qquad S_\infty = \frac{a}{1-r} = \frac{2}{1/2} = 4$

c) $(2x-3)(x+5)=0 \qquad x=3/2,\ -5$

d) $d = \frac{|30-8-7|}{\sqrt{3^2+4^2}} = \frac{15}{5} = 3$

e) $\frac{1}{2\sqrt{2}-1} \times \frac{2\sqrt{2}+1}{2\sqrt{2}+1} = \frac{2\sqrt{2}+1}{7}$

f) $l = r\theta = 4 \times \frac{5\pi}{18} = 3.5$

Q2 a) i/ $y' = e^x 2x + x^2 e^x$

$= e^x(2x+x^2) = xe^x(2+x)$

a) ii/ $y = \tan x \qquad y' = \sec^2 x$

b) i/ $\frac{1}{3}\int \frac{3x^2+3}{x^3+3}\,dx = \frac{1}{3}\ln(x^3+3)+C$

b) ii/ $\int_1^2 \frac{2}{1-3x}\,dx = -\frac{2}{3}\int_1^2 \frac{-3}{1-3x}\,dx$

$= -\frac{2}{3}\Big[\ln(1-3x)\Big]_1^2 = \left|-\frac{2}{3}\left(\ln|-5| - \ln|(-2)|\right)\right|$

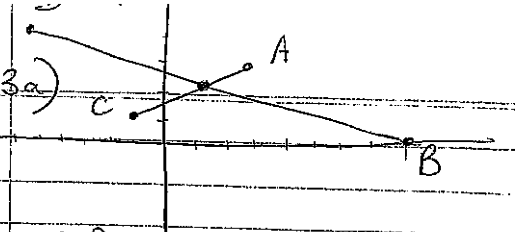
$= \frac{2}{3}\ln\frac{5}{2}$

c) $y' = -\frac{4}{x^2}$ at $x=1, \quad y'=-4$

$y-4 = -4(x-1)$

$y = -4x+4+4 \qquad y = -4x+8$

Q3 a)



$m(AC) = \frac{1}{2} \qquad m(DB) = -\frac{2}{7}$

Equ AC. $\qquad$ Equ DB

$x-2y=-3 \qquad y = -\frac{2x}{7} + 2\,2/7$

Solving Sim.

$\left.\begin{aligned} x-2y&=-3 \\ 2x+7y&=16 \end{aligned}\right\} \qquad \begin{aligned} 2x-4y&=-6 \\ 2x+7y&=16 \end{aligned}$

$\therefore 11y=22 \quad y=2, \quad x=1 \qquad (1,2)$

b) $T_{30} = 5+(29)6 \qquad S_{30} = \frac{30}{2}(5+179)$

$= 179 \qquad = 2760$

c) $S_\infty = \frac{a}{1-r} = 7$ —— ①

i/ $S_2 = \frac{a(1-r^2)}{1-r} = \frac{a(1-r)(1+r)}{(1-r)} = a(1+r) = \frac{48}{7}$ ②

$a = 7(1-r)$ sub in ② $\qquad 7(1-r)(1+r) = \frac{48}{7}$

$49(1-r)(1+r) = 48$

$49(1-r^2) = 48 \qquad 49-49r^2 = 48$

$1-49r^2 = 0$

$r^2 = \frac{1}{49} \qquad r = \pm\frac{1}{7}$

ii/ $a = 7(1-r)$

$a = 7\left(1-\frac{1}{7}\right) = 7 \times \frac{6}{7} \qquad a=6$

4a) $\cos^2\theta = \frac{1}{2}$ $\cos\theta = \pm\frac{1}{\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}}$ $\cos\theta = -\frac{1}{\sqrt{2}}$

$\theta = 45°, 315°$ $\theta = 135°, 225°$.

b) $m^2 + 5m - 14 = 0$.

$(m-2)(m+7) = 0$ $m = 2$ or -7

$x^2 = 2$ $\underline{x = \pm\sqrt{2}}$

$x^2 = -7$ no solns.

c) i/ $l = r\theta$ $5 = 4\theta$ $\underline{\theta = \frac{5}{4}}$ rads

ii/ $A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= 8\left(\frac{5}{4} - \sin\frac{5}{4}\right) = \underline{2.408}$

d) $y = \frac{1}{x^2}$ $y' = -\frac{2}{x^3}$ at $x = -1$, $y' = 2$

Eqn $y - 1 = 2(x+1)$ $\underline{y = 2x+3}$

5a) $\Delta = 0$ $(-8k)^2 - 4(5k+1)(3k) = 0$.

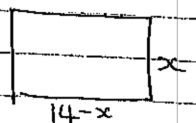
$64k^2 - (20k+4)(3k) = 0$.

$64k^2 - 60k^2 - 12k = 0$

$4k^2 - 12k = 0$.

$4k(k-3) = 0$.

$\underline{k = 0 \text{ or } 3}$

b) 

$A = x(14-x) = 14x - x^2$

$A' = 14 - 2x$

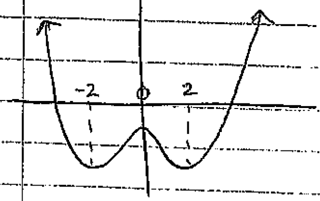
for sp's $A' = 0$, $14 - 2x = 0$, $\underline{x = 7}$

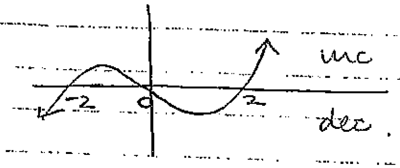
Max Area $= 7(14-7) = \underline{49\,u^2}$

5c) $f(x) = x^4 - 8x^2 - 3$. $f'(x) = 4x^3 - 16x$

find sp's. $= 4x(x^2 - 4)$

$= 4x(x-2)(x+2)$

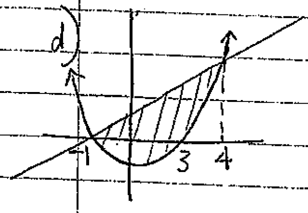




Use either to find where function is inc or dec

$f'(x) > 0$ $x > 2$, $-2 < x < 0$

$f'(x) < 0$ $x < -2$, $0 < x < 2$

d) 

$A = \int_{-1}^{4} \left[(x+1) - (x^2 - 2x - 3)\right] dx$

$= \int_{-1}^{4} (-x^2 + 3x + 4)\,dx = \left[\frac{-x^3}{3} + \frac{3x^2}{2} + 4x\right]$

$= \underline{\frac{125}{6}\,u^2}$

Q6a) $3^x = 10$ take logs $x\log 3 = \log 10$

$x = \frac{1}{\log 3} = 2.10$

b) i/ $f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3)$

for sp's $f'(x) = 0$ $\underline{x = 0, 3}$ $(0,0)$ $(3,-27)$

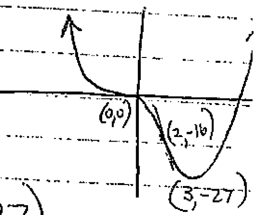
ii/ $f''(x) = 12x^2 - 24x = 12x(x-2)$

For pts of infl $f''(x) = 0$, $x = 0, 2$

$\therefore (0,0)$ horiz pt of infl.

$(2,-16)$ non-horiz pt of infl

At $x = 3$, $y''(x) > 0$ $\therefore$ Min TP at $(3,-27)$



6c) $x^2+2x+(1)^2+y^2-4y+(-2)^2=4+1+4$

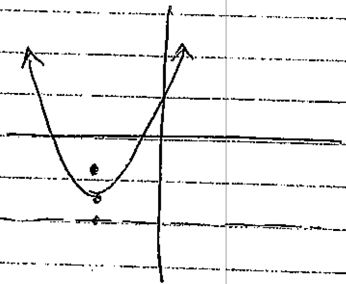
$(x+1)^2+(y-2)^2=9$

$\therefore$ Radius 3, $(-1,2)$ centre

Q7 $y=x^2+6x+(3)^2-9+6$

$x^2+6x+(3)^2=y+9-6$

$(x+3)^2=1(y+3)$



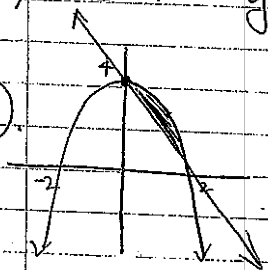
i/ $\therefore$ Vertex $(-3,-3)$

$a=\frac{1}{4}$

ii/ Focus $(-3,-2\frac{3}{4})$

iii/ Dir. $y=-3\frac{1}{4}$

b)



$V=\pi\int_0^2\left[(4-x^2)^2-(4-2x)^2\right]dx$

$=\pi\int_0^2(x^4-12x^2+16x)\,dx$

$=\pi\left[\frac{x^5}{5}-4x^3+8x^2\right]_0^2$

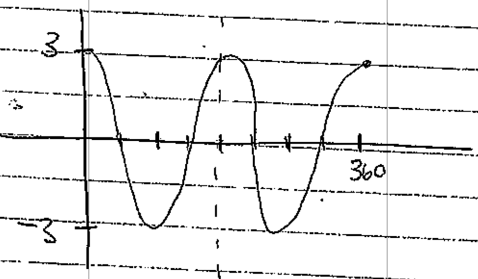
$=\frac{32\pi}{5}$

c) $y=3\cos2\theta$

i/ Ampl $=3$

ii/ period $=\frac{2\pi}{2}=\pi$

iii/



8a) $y=\ln\left(\frac{x-1}{x+1}\right)$ $y=\ln(x-1)-\ln(x+1)$

$y'=\frac{1}{x-1}-\frac{1}{x+1}$ at $x=3$, $y=\ln\frac{1}{2}$

$y'=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$ $\therefore$ Grad normal $=-4$

$\therefore$ Equ of normal : $y-\ln\frac{1}{2}=-4(x-3)$

$y=-4x+12+\ln\frac{1}{2}$

b)

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
y	e^{-2}	e^{-3}	e^{-4}	e^{-5}	e^{-6}

$h=\frac{1}{2}$

$A=\frac{1}{6}\left(e^{-2}+4e^{-3}+2e^{-4}+4e^{-5}+e^{-6}\right)$

$\doteqdot 0.067$

c) $4\cos^2\theta+3\sin\theta-4=0$ but $\cos^2\theta=1-\sin^2\theta$

$4-4\sin^2\theta+3\sin\theta-4=0$

$4\sin^2\theta-3\sin\theta=0$

$\sin\theta(4\sin\theta-3)=0$ $\therefore\sin\theta=0$ or $\sin\theta=\frac{3}{4}$

$\therefore\theta=0°, 180°, 360°$

$\theta=48.6°, 131.4°$

d) $y=\sqrt{6+x}\sqrt{3-2x}$

$u=(6+x)^{\frac{1}{2}}$

$u'=\frac{1}{2}(6+x)^{-\frac{1}{2}}=\frac{1}{2\sqrt{6+x}}$

$v=\sqrt{3-2x}$

$v'=\frac{1}{2}(3-2x)^{-\frac{1}{2}}\cdot-2$

$=-\frac{1}{\sqrt{3-2x}}$

$y'=vu'+uv'$

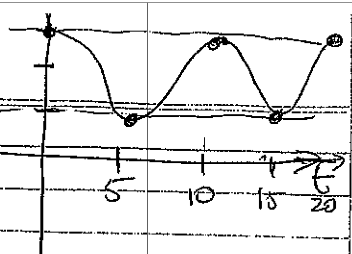
$=\frac{\sqrt{3-2x}}{2\sqrt{6+x}}+\frac{\sqrt{6+x}\cdot-1}{\sqrt{3-2x}}$

$=\frac{\sqrt{3-2x}}{2\sqrt{6+x}}-\frac{\sqrt{6+x}}{\sqrt{3-2x}}$

$=\frac{-12-2x+3-2x}{2\sqrt{3-2x}\sqrt{6+x}}$

$=\frac{-4x-9}{2(3-2x)^{1/2}(6+x)^{\frac{1}{2}}}$

Q9 a) $x = 4 + 2\cos\frac{\pi}{5}t$



i/
* Ampl = 2
* graph moves up 4. (Range $2 \le y \le 6$)
* period $= \frac{2\pi}{\pi/5} = 10$

ii/ $v = \dot{x} = -\frac{2\pi}{5}\sin\frac{\pi}{5}t$

When $v = 0$, $\sin\frac{\pi}{5}t = 0$

(SP's)

$\therefore \frac{\pi}{5}t = 0, \pi, 2\pi, 3\pi, 4\pi,$

$t = 0, 5, 10, 15, 20$

b) $P = P_0 e^{kt}$

i/ When $t = 15$, $P = 2P_0$ $\quad 2P_0 = P_0 e^{k \times 15}$

$2 = e^{15k}$ Take logs. $\log 2 = 15k \log e$

$\therefore k = \frac{\log 2}{15}$

ii/ Find when $P = 4P_0$

$4P_0 = P_0 e^{\frac{\log 2}{15}t}$ $\quad 4 = e^{\frac{\log 2}{15}t}$

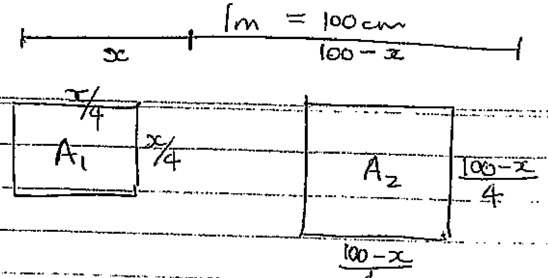
Take logs $\quad \log 4 = \frac{\log 2}{15}t \log e$

$t = \frac{15\log 4}{\log 2} \doteq 30$ yrs

iii/ $P = 30\,000\,e^{35\left(\frac{\log 2}{15}\right)}$

$= 151\,190$

Q10 a)



$A_1 + A_2 = 325$

$\frac{x^2}{16} + \frac{(100-x)^2}{16} = 325$

$x^2 + 10000 - 200x + x^2 = 5200$

$2x^2 - 200x + 4800 = 0$

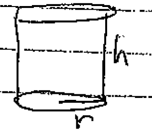
$(\div 2)$ $\quad x^2 - 100x + 2400 = 0$

$(x-40)(x-60) = 0$

$\therefore x = 40$ or 60

$\therefore P_1 = 40$ cm $\quad P_2 = 60$ cm

b)



$V = 54\pi$ cm^3

To find h $\quad \pi r^2 h = 54\pi$

$h = \frac{54}{r^2}$

i/ $\therefore$ Surface Area $= 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\pi r \times \frac{54}{r^2}$

$A = 2\pi r^2 + \frac{2\pi \times 54}{r} = 2\pi r^2 + \frac{108\pi}{r}$

ii/ $A' = 4\pi r - \frac{108\pi}{r^2}$

For sp's $A' = 0$, $\quad \frac{108\pi}{r^2} = 4\pi r$

$r^3 = 27 \quad \therefore r = 3$

$h = \frac{54}{9} = 6$